RISK-AVERSE SUPPLIER AND RETAILER'S FINANCING DECISION

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ABSTRACT

In this paper, by setting up a single period Stackelberg game supply chain model with one supplier as leader and one retailer as follower, we study the capital constrained retailer' procurement and financing problem when the supplier has different risk preference. When the supplier is risk neutral, we find the supplier induces the retailer to choose trade credit. When the supplier is risk-averse, we find if the supplier is risk averse very much, she induces the retailer to choose bank loan; if the supplier's risk preference is higher than some specific value, she prefers to bear the credit risk and induces the retailer to choose trade credit; Otherwise, in a numerical experiment, we observe the portfolio of bank loan and trade credit is optimal.

INTRODUCTION

The operation of supply chain is a continuous flow of products, information and capital. The continuity implies that there is sufficient working capital available. Most prior studies inevitably assume that firms have no capital constraints in operation, and paid more attention on the coordination problem of products and information. In practice, however, there are many situations in which suppliers or retailers are facing capital constraints and therefore limited in their operational decisions, especially to SMEs. We study how suppliers and retailers make decisions when retailers are capital constrained. When a retailer is facing liquidity constraints in procuring, he is in need of short term financing to execute his procuring action. An obvious financing source is bank loan, but relative to big enterprisers, SMEs usually have limited access to bank credit as a result of their own characteristics. Supplier financing, or trade credit, is another solution of short term financing [1]. [2] found that 73% of European large corporate are looking to extend payment terms with their supplies in 2007. [3] found, 63% of companies in the UK are trying to extend credit, compared with 48% in Germany. For example, Wal-Mart uses trade credit as a preferable financing source. Wal-Mart had $28.8 billion accounts payable amounting to 75% of its total inventory in January, 2009. In addition, HP, IBM, Sony etc., all provide trade credit to their distributors or resellers. Especially for HP, the financing of its products could equal up to 100% of the value of the borrower's inventory [4].

Bank loan and trade credit differ in one main aspect, i.e., the party who takes the credit risk, the bank or the supplier. Due to the uncertainty of demand, it is possible that the retailer cannot fully pay off the debt. This difference brings us the research question: when the supplier has different risk preference, how does he induce the retailer to choose financing solution?

The literature on operations and finance interface and supply chain contracts are closed related to this paper. There are numerous literatures on supply chain contracts. [5] has done an excellent survey, and our work is complementary to this direction by taking financial constraints into consideration. Recently, the literatures on operation and finance interface are arising, including important papers by [6] [7] [8]. The papers [4] [9] [10] [11] are most related to this paper. These working papers all study a single period Stackelberg game supply chain model with a capital constrained retailer. This paper and [11] both can be seen as extensions of [9] [10]. The difference is they introduce distress cost, but we suppose the supplier has different risk preference.

MODEL DESCRIPTION

We study a single period Stackelberg game supply chain model with one supplier as leader and one retailer as follower. The supplier has deep pocket, risk neutral or risk averse. The retailer is risk neutral, and has initial cash B_r to procure products from the supplier. At time t_0, the supplier offers a trade credit contract (w, r_s), in which w is the wholesale price and r_s is an early payment discount, and the retailer orders q. At time t_1, the demand realizes. Let c be unit cost, and p be retail price. The financial market provides financial service for the retailer. We represent the terms of bank loan in the form of triplet $(L, \mathcal{L}, r_b)$, in which L is the amount that the retailer borrows when the procurement contract is signed, r_b is the nominal interest rate, and $\mathcal{L}$ is the equity that the retailer has to repay the loan plus the interest after demand is realized. The retailer is required to pay $\min(\mathcal{L}, L(1+r_b))$.
We assume the financial market is competitive as [10] does. We assume risk free rate $r_f = 0$. We also assume demand D with distribution $F(x)$ satisfies the following properties: it has a smooth density $f(x) > 0$ in the support $(0, +\infty)$, and has a finite mean; it's failure rate $h(x) = \frac{f(x)}{\overline{F}(x)}$ is weakly increasing in x such that $F(x) < 1$; $h(x)$ is a convex or constant function; $\frac{-1}{h(x)}$ is a concave or constant function [9].

MODEL ANALYSIS

The Retailer's Optimal Ordering and Financing Policy

To finance inventory, the retailer uses initial capital and two financing solutions, bank credit or trade credit. We define the default threshold d_b for the bank loan, d_s for the trade credit. Since bank loan should be paid off first, $d_s \geq d_b \geq 0$. Let $q = q_0 + q_1$, where q_0 is the quantity paid by delivery, and q_1 is the quantity paid after sale. The retailer's expected profit function is

$$\pi_r(q_0, q_1) = E[(p\min(D, q) + B_r + \frac{pd_b}{1+r_b} - \frac{wq_0}{1+r_s} - pd_s)^+] - B_r. \tag{1}$$

Since we have assumed the financial market is competitive, so d_s, d_b, q_0, and q_1 satisfy two equations:

$$(\frac{wq_0}{1+r_s} - B_r)^+ = p\int_0^{d_b} \overline{F}(x)dx, \tag{2}$$

$$wq_1 = p(d_s - d_b). \tag{3}$$

Lemma 1. *The retailer's strategy of financing inventory is that: initial capital* B_r *is the retailer's first choice; if* $r_s > 0$*, bank loan is the second, and the trade credit is the last choice when* B_r *is used up; if* $r_s = 0$*, trade credit is the second choice, and bank loan will not be chosen.*

Following the Lemma 1, the retailer's expected profit function can be rewritten as

$$\pi_r(q_0, q_1) = \begin{cases} p\int_0^{q_0} \overline{F}(x)dx - \frac{wq_0}{1+r_s}, & d_b = d_s = 0 \\ p\int_{d_b}^{q_0} \overline{F}(x)dx - B_r, & d_b = d_s > 0 \\ p\int_{d_s}^{q} \overline{F}(x)dx - B_r, & d_s > d_b > 0 \end{cases} \tag{4}$$

Take the first order derivative, we get

$$(\pi_r(q_0, q_1))'_q = \begin{cases} p\overline{F}(q_0) - \frac{w}{1+r_s}, & d_b = d_s = 0 \\ p\overline{F}(q_0) - \frac{w}{1+r_s}, & d_b = d_s > 0 \\ p\overline{F}(q) - w\overline{F}(d_s), & d_s > d_b > 0 \end{cases} \tag{5}$$

Proposition 1. $\pi_r(q_0, q_1)$ *is unimodal, and there exists an unique* $(q_0(w, r_s), q_1(w, r_s))$ *maximizing* $\pi_r(q_0, q_1)$*. The retailer's optimal ordering and financing policy is as follows:*

1) If $B_r \geq \frac{w\hat{q}_0(w,r_s)}{1+r_s}$*,* B_r *is large enough that the retailer can use* B_r *to achieve optimal order* $q_0(w, r_s) = \hat{q}_0(w, r_s)$*, and* $q_1(w, r_s) = 0$*, where* $\hat{q}_0(w, r_s) \geq 0$ *satisfies* $p\overline{F}(\hat{q}_0) - \frac{w}{1+r_s} = 0$*;*

2) if $\frac{w\hat{q}_0(w,r_s)}{1+r_s} - p\int_0^{\overline{F}^{-1}(\frac{1}{1+r_s})} \overline{F}(x)dx \leq B_r < \frac{w\hat{q}_0(w,r_s)}{1+r_s}$*,* B_r *is not so large that the retailer should use* B_r *and bank loan to achieve optimal order* $q_0(w, r_s) = \hat{q}_0(w, r_s)$ *,* $q_1(w, r_s) = 0$ *, and* $d_b(w, r_s) = d_s(w, r_s)$ *is determined by the equation (2);*

3) if $0 \leq B_r < \frac{w\hat{q}_0(w,r_s)}{1+r_s} - p\int_0^{\overline{F}^{-1}(\frac{1}{1+r_s})} \overline{F}(x)dx$*, the retailer uses* B_r*, bank loan and trade credit to finance inventory,* $q(w, r_s) = q_0(w, r_s) + q_1(w, r_s)$ *,* $q_0(w, r_s) = \frac{1+r_s}{w}(B_r + p\int_0^{\overline{F}^{-1}(\frac{1}{1+r_s})} \overline{F}(x)dx)$ *,* $d_b = \overline{F}^{-1}(\frac{1}{1+r_s})$*,* $d_s(w, r_s)$ *and* $q_1(w, r_s)$ are uniquely determined by $wq_1(w, r_s) = p(d_s - d_b)$ *and* $p\overline{F}(q_0(w, r_s) + q_1(w, r_s)) = w\overline{F}(d_b + \frac{wq_1(w,r_s)}{p})$*.*

Proposition 1 tells us that given the supplier's trade credit contract, the retailer responses uniquely, so as the Stackelberg leader, the supplier can use contract to control the retailer's ordering and financing decision.

The Risk-Neutral Supplier's Optimal Contract

The supplier offers a contract(w, r_s), his profit function is

$$v_s(w, r_s) = \begin{cases} (\frac{w}{1+r_s} - c)q_0(w, r_s), & case1, case2 \\ \min(wq_1(w, r_s), p(\min(q_0(w, r_s) + q_1(w, r_s), D) - d_b(w, r_s))^+) & \\ + \frac{wq_0(w,r_s)}{1+r_s} - c(q_1(w, r_s) + q_0(w, r_s)), & case3 \end{cases} \tag{6}$$

and his expected profit function is

$$\pi_s(w,r_s)=\begin{cases}(\frac{w}{1+r_s}-c)q_0(w,r_s), & case1\\ p\int_0^{d_b(r_b)}\overline{F}(x)dx-cq_0(w,r_s)+B_r, & case2\\ p\int_0^{d_s(w,r_s)}\overline{F}(x)dx-cq(w,r_s)+B_r, & case3\end{cases} \tag{7}$$

where case1, case2 and case3 are the cases of the retailer's different best responses.

Lemma 2. The supplier's expected profit function $\pi_s(w,r_s)$ can be rewritten as a function of q in case1, a function of d_s in case2, and a function of (d_s,d_b) in case3.

Proposition 2. *There exists an unique q_0^* maximizing $\pi_s(q_0)$, and there exists an unique $(d_s^*,0)$ maximizing $\pi_s(d_s,d_b)$. If $\pi_s(q_0^*)\geq\pi_s(d_s^*,0)$, the supplier chooses case1, otherwise, she chooses case3.*

Proposition 2 shows there exists a Stackelberg equilibrium (SE) respectively in case1 and case3, and as leader the supplier chooses the better one from her perspective under different market condition. In addition, we find the risk-neutral supplier never induces the retailer to choose bank loan.

The Risk-Averse Supplier's Optimal Contract

In this section we want to see when the supplier is risk-averse whether the SE is changed. We assume the supplier is risk-averse is reasonable. First, risk-neutral implies the supplier can hedge risk in a financial market, but in reality she can not. Second, we can observe in practice that a bigger supplier is usually more cares about risk relative to profit, and she is usually willing to tradeoff lower expected profit for downside protection against possible losses in making decision. Considering the small retailer, his potential losses are bounded by B_r, so he orders aggressively. It is intuitive to assume the retailer is risk-neutral.

The risk measurement plays a crucial role in optimization under uncertainty, and the risk-averse supplier is to maximize Conditional Value-at-Risk (CVaR). There are several advantages to use CVaR approach both in theory and in application. First, CVaR is a coherent measurement of risk, and is known to have better properties than Value-at-Risk (VaR). Second, it avoids penalizing equally the desirable upside and the undesirable downside outcomes. Third, it is easily to compute and thus obtain a closed-form solution in the first step, which proposes an intuitive explanation for the impact of risk-aversion parameter. Last, a risk neutral model of this problem is a special case of this model based on CVaR approach. The supplier's profit $v_s(w,r_s)$ is random. The CVaR approach ignores the profit beyond the specified level, and focuses on the average profit from the lower quantiles. So, the supplier is to solve the following optimization problem [12]

$$\max_{w,r_s\geq 0} C_\beta v_s(w,r_s) \tag{8}$$

Where

$$C_\beta v_s(w,r_s)=\max_{\alpha\in R}\left\{\alpha+\frac{1}{\beta}E[(v_s(w,r_s)-\alpha)^-]\right\},$$

where α is a real number, and $\beta(0<\beta<1)$ is a risk-aversion parameter.

Proposition 3. *1) In case1, the risk-averse supplier' problem is the same as the risk-neutral supplier's; 2) In case 3, there exists an β_1, if $\beta\leq\beta_1$, the supplier prefers the retailer to choose bank loan as the unique financing solution; there exists an β_3, when $\beta\geq\beta_3$, she prefers trade credit; when $\beta=1$, the* *risk-averse supplier's problem degenerates into the risk-neutral supplier's problem.*

Proposition 3 tells us that the supplier's preference level has impact on the retailer's optimal ordering and financing strategy. It also shows the existence of β_1 and β_3, which are the thresholds for the retailer to change optimal strategy. Since we can not get more analytical results about the equilibrium, we want to conduct a set of numerical experiments to get some insights. We choose the outcomes in the range $\beta \geq F(d_s) \geq F(d_b)$, which highlights the most important features and are enough to for us to figure out the equilibrium. Let (d_s^3, d_b^3) denote the supplier's optimal strategy in this range.

To perform the experiment, we use a Weibull distribution $\overline{F}(x) = \exp(-(\frac{x}{\lambda})^k)$ to model demand uncertainty where $\lambda > 0$, $k \geq 2$, and $f(0) = 0$. Weibull distribution satisfies properties assumed previously, and it simplifies the computation. We set retail price $p = 100$, unit cost $c = 50$, $(\lambda, k) = (1000, 2)$, $0 < \beta < 1$, and $0 < B_r \leq p\overline{F}(\bar{q}_1)\bar{q}_1 = 36386.3$ to guarantee the retailer has default risk. Table 1 summarizes the results of the experiment.

Table 1: The optimal solution (d_b^3, d_s^3) in $\beta > F(d_s) \geq F(d_b)$

$Br = 36000, \beta$ (d_b^3, d_s^3)	$\beta_3 = 0.004$ (0,3.76)	0.6 (0,4.28)	0.99 (0,4.28)			
$Br = 30000, \beta$ (d_b^3, d_s^3)	$\beta_1 = 0.004$ (63.95,63.95)	$\beta_3 = 0.059$ (0,62.81)	0.6 (0,72.26)	0.99 (0,72.74)		
$Br = 20000, \beta$ (d_b^3, d_s^3)	0.01 (100.25,100.25)	$\beta_1 = 0.027$ (165.36,165.36)	$\beta_3 = 0.156$ (0,169.14)	0.6 (0,194.48)	0.99 (0,198.05)	
$Br = 10000, \beta$ (d_b^3, d_s^3)	0.01 (100.25,100.25)	$\beta_1 = 0.07$ (270.31,270.31)	$\beta_3 = 0.253$ (0,289.18)	0.6 (0,337.97)	0.99 (0,350.01)	
$Br = 5000, \beta$ (d_b^3, d_s^3)	0.01 (100.25,100.25)	$\beta_1 = 0.101$ (324.95,324.95)	$\beta_2 = 0.298$ (322.36,325.81)	$\beta_3 = 0.301$ (0,357.90)	0.6 (0,429.27)	0.99 (0,451.48)
$Br = 1000, \beta$ (d_b^3, d_s^3)	0.01 (100.25,100.25)	$\beta_1 = 0.128$ (370.09,370.09)	$\beta_2 = 0.323$ (366.57,371.67)	$\beta_3 = 0.35$ (0,427.12)	0.6 (0,538.62)	0.99 (0,584.76)
$Br = 500, \beta$ (d_b^3, d_s^3)	0.01 (100.25,100.25)	$\beta_1 = 0.132$ (375.83,375.83)	$\beta_2 = 0.326$ (372.13,377.56)	$\beta_3 = 0.354$ (0,435.52)	0.6 (0,561.81)	0.99 (0,617.57)
$Br = 10, \beta$ (d_b^3, d_s^3)	0.01 (100.25,100.25)	$\beta_1 = 0.135$ (381.49,381.49)	$\beta_2 = 0.328$ (381.09,381.69)	$\beta_3 = 0.371$ (0,452.84)	0.6 (0,596.19)	0.99 (0,692.63)

In table 1, we observe $\beta_1 = F(\hat{d}_b)$, and when $\beta \leq \beta_1$, the supplier induces the retailer to choose bank loan as the unique financing solution. We also abserve β_3, and β_3 is decreasing in B_r. When B_r is relative small, we observe β_2. If $\beta_1 \leq \beta < \beta_2$, $(d_b^*, d_s^*) = (\hat{d}_b, \hat{d}_b)$, i.e., the supplier still induces the retailer to choose bank loan as the unique financing solution. If $\beta_2 \leq \beta < \beta_3$, the portfolio of bank loan and trade credit is optimal. We find when B_r is relative bigger, β_2 disappears, which implies the supplier never chooses the portfolio as the equilibrium.

Conclusions and Further Considerations

In this paper, we set up a single period supply chain model with one supplier and one retailer. The supplier has deep pocket, risk neutral or risk averse. The retailer is risk neutral, and has working capital constraint. We study how the retailer chooses financing solution when the supplier has different risk preference. We first study the retailer's financing decision when the supplier is risk neutral. We find the supplier induces the retailer to choose trade credit as the only financing scheme, and bears the credit risk. This finding is consistent with the conclusion in [9] [10]. Next we study the retailer's financing decision when the supplier has different risk preference. We find when the supplier is risk averse very much, he induces the retailer to choose bank loan; when the supplier's risk preference level is higher than some specific value, he prefers to bear the credit risk and induces the retailer to choose trade credit as the unique financing solution; Otherwise, from the numerical experiment, we observe the portfolio of bank loan and trade credit, and this finding is more intuitive.

One possible extension is to address the information asymmetry issue from this paper or from [9] [10]. Our model still assumes the retailer's initial capital is common shared information, however, in reality, this is not true. In practice, the retailer may misrepresent his initial capital to obtain a more favorable contract and to improve his profit, so how the supplier induces the retailer to choose financing solution under information asymmetrical condition is interesting question.

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