PRODUCT-MIX LINEAR PROGRAMMING MODELS WITH FINANCING COSTS

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ABSTRACT

Accountants provide important information for product-mix optimization problems such as selling prices and variable costs for direct materials, direct labor, and manufacturing overhead. With additional information for production and selling constraints, accountants solve Linear Programming (LP) models by optimally allocating resources such that profits are maximized. However, financing costs are seldom explicitly modeled in determining an initial "optimal" product-mix. The Virtual Company case illustrates how interest costs for borrowings and debt covenants can be integrated into typical product-mix LP models.

INTRODUCTION

Managerial and cost accounting courses do not usually present financing costs when modeling a product-mix problem (Hilton et al., 2003; Horngren et al., 2006). Instead, what is typically presented are data for selling prices and variable manufacturing costs, production constraints for direct materials, direct labor, or machine usage, and units to be sold. Yet, from an initial "optimal" product-mix solution, managers will often consider financing costs that lead to changes in an optimal product-mix. For example, expected costs of interest expense on borrowings and debt covenants may significantly change the product mix. Accounting students should learn how to integrate financing costs into an LP model for a product-mix decision.

A product-mix problem is commonly presented as a Linear Programming (LP) model in which scarce resources are allocated such that an objective function is optimized. Contribution margin maximized or costs minimized are typical objective functions within the accounting curriculum (Horngren et al., 2006). With spreadsheets having add-ins for linear programming (e.g., Solver in EXCEL), product-mix problems taught in accounting can easily include financing costs for many products. The following Virtual Company case is an example of adding financing considerations to an LP model for product-mix decisions.

INTEGRATING FINANCING CONSIDERATIONS FOR AN LP MODEL

Initial Product-Mix Model Without Financing Considerations

Virtual Company is a manufacturer of four custom appliances. Panel A of Table 1 identifies the selling price, variable manufacturing costs and contribution margin for Products 1, 2, 3 and 4. In Panel A, the number of machine hours required for each product using Machines A, B and C, and the total hours available for each of the three machines over the three-month period are also listed. For example, Product 1 uses 130 hours of Machine 1, 170 hours of Machine 2, and 210 hours of Machine 3. The total number of hours available on Machines 1, 2 and 3 are 10,000, 12,000, and 15,000. Over the three-

month period, Product 1 has a sales requirement of at least 8 units, Product 2 of 20 units, Product 3 of 12 units, and Product 4 of 8 units.

Given the input data, Panel B displays the initial LP solution of 10 units of Product 1, 45 units of Product 2, 12 units of Product 3 and 10 units of Product 4. This initial solution will maximize contribution margin at $132,000, with production and sales requirements met. The solution was set to have nonnegative integers.

Insert Table 1 About Here

Adding Financing Requirements

Financing requirements for the four-product LP problem will focus on cash borrowings and a bank debt covenant. The model assumes variable manufacturing costs will occur 3 months before the collection of revenues, and funds can be borrowed at an annual interest rate of 16%; therefore, a 4% interest rate is used for interest expense.

Over the three-month period, borrowings are equal to the variable manufacturing costs less funds of $150,000 set aside for production. Funds used for fixed cash expenses and related interest expenses will not differ between alternatives for a mix of products. In other words, an LP model considers costs and revenues that vary with the product mix.

The equations 1 and 2 are for B borrowings, where P1, P2, P3 and P4 are the number of Product 1, Product 2, Product 3, and Product 4 units produced. Variable manufacturing costs from Panel A of Table 1 for Products 1, 2, 3, and 4 are $6,000, $5,000, $4,000 and $5,000, respectively. Total borrowings B is equal to the variable manufacturing costs for each product less the $150,000 set aside for production.

$$6{,}000*P1 + 5{,}000*P2 + 4{,}000*P3 + 5{,}000*P4 - 150{,}000 = B \quad (1)$$

or

$$6{,}000*P1 + 5{,}000*P2 + 4{,}000*P3 + 5{,}000*P4 - B = 150{,}000 \quad (2)$$

When the initial solution of 10 units of P1, 45 units of P2, 12 units of P3 and 10 units of P4 are entered into the above equation, B borrowings is equal to $233,000 ($60,000 + $225,000 + $48,000 + $50,000 - $150,000).

The debt covenant specifies that the total amount for B borrowings and its interest must not exceed 30% of the combined related account receivables and $150,000. The 16% annual rate converts to a 4% rate over the three-month period or interest expense of .04B. The selling prices for Products 1, 2, 3, and 4 from Panel A of Table 1 are $8,000, $7,000, $5,000 and $6,000. The following equations 3, 4, and 5 are for the debt covenant.

$$B + .04B \leq .30(8{,}000*P1 + 7{,}000*P2 + 5{,}000*P3 + 6{,}000*P4 + 150{,}000) \quad (3)$$

$$1.04B/.30 \leq 8{,}000*P1 + 7{,}000*P2 + 5{,}000*P3 + 6{,}000*P4 + 150{,}000 \quad (4)$$

$$-8{,}000*P1 - 7{,}000*P2 - 5{,}000*P3 - 6{,}000*P4 + 1.04B/.30 \leq 150{,}000 \quad (5)$$

From the last equation above, the initial solution of 10 units of P1, 45 units of P2, 12 units of P3 and 10 units of P4 requires borrowings of $233,000, while generating receivables of $515,000 ($80,000 + $315,000 + $60,000 + $60,000). The debt covenant amount of $807,733 (233,000*1.04/.30) exceeds

the receivables of $515,000 by $292,733, which violates the $150,000 constraint (see abbreviated LP model in Panel C of Table 1). Hence, the initial LP solution does not meet the debt covenant, even though it does meet the production and sales requirements.

Product Mix For Financial, Production and Sales Considerations

Financing requirements must recognize the available $150,000 and the 30% debt limitation placed on related accounts receivables and the $150,000. Given these additional requirements, the LP solution is found in Panel D of Table 1. The optimal solution where all production, sales and financing requirements are met is the production of 8 units of P1, 35 units of P2, 13 units of P3, and 8 units of P4. This reflects a shift towards less costly units when also considering the cost of borrowings. Substantial borrowings of $165,000 are needed which incurs interest costs of $6,600 ($165,000*.04). Consequently, the total contribution margin for this solution is $100,400, which is much lower than the initial solution contribution margin of $132,000. The explicit inclusion of financing considerations significantly changed the optimal product mix and profits. This example for including financing costs substantiates what managers have intuitively recognized – that cost of borrowing funds must be considered in product-mix decisions.

OTHER BENEFITS OF MODELING FINANCING COSTS

With an LP model that includes financing costs, techniques to reduce the total cost of production, selling, and financing can be examined. Management could examine strategies to improve profitability and cash flow that focus on finding lower interest rates, less restrictive debt covenants, collecting deposits on orders, or offering customer discounts for quick payments on receivables. Techniques for integrating relevant costs for production, sales and financing of products are being facilitated by enhanced spreadsheet capabilities.

References

[1] Hilton, Ronald W., Michael W. Maher and Frank H. Selto, *Cost Management: Strategies for Business Decisions*, McGraw-Hill/Irwin, New York, 2003.

[2] Horngren, Charles T., George Foster and Srikant M. Datar, *Cost Accounting: A Managerial Emphasis*, Prentice-Hall, Upper Saddle River, New Jersey, *2006.*

TABLE 1: LP MODELING FOR FINANCING COSTS

Panel A – Input Data

	Prod. 1	**Prod. 2**	**Prod. 3**	**Prod. 4**	**Borrow.**	**Total**		**Constraint**
Selling price	8,000	7,000	5,000	6,000				
Variable manuf.	6,000	5,000	4,000	5,000				
Contrib. margin	2,000	2,000	1,000	1,000				
Production:								
Machine 1	130	120	100	110			<=	10,000
Machine 2	170	150	130	140			<=	12,000
Machine 3	210	180	150	170			<=	15,000
Sales:								
Product 1							>=	8
Product 2							>=	20
Product 3							>=	12
Product 4							>=	8

Panel B – Initial Solution

Decision	**10**	**45**	**12**	**10**				
Contrib. margin	20,000	90,000	12,000	10,000		**132,000**		
Production:								
Machine 1	1,300	5,400	1,200	1,100		9,000	<=	10,000
Machine 2	1,700	6,750	1,560	1,400		11,410	<=	12,000
Machine 3	2,100	8,100	1,800	1,700		13,700	<=	15,000
Sales:								
Product 1	10					10	>=	8
Product 2		45				45	>=	20
Product 3			12			12	>=	12
Product 4				10		10	>=	8

Panel C – Violated Debt Covenant With Initial Solution

Decision	**10**	**45**	**12**	**10**	**233,000**			
Contrib. margin	20,000	90,000	12,000	10,000	-9,320	122,680		
Production:	No Change From Panel B Above							
Sales:	No Change From Panel B Above							
Financing:								
Cash borrowings	60,000	225,000	48,000	50,000	**-233,000**	**150,000**	**=**	**150,000**
Debt covenant	-80,000	-315,000	-60,000	-60,000	807,733	**292,733**	**<=**	**150,000**

Panel D – Optimal Solution for Production, Sales and Financing

Decision	**8**	**35**	**13**	**8**	**165,000**			
Contrib. margin	16,000	70,000	13,000	8,000	**-6,600**	**100,400**		
Production:								
Machine A	1,040	4,200	1,300	880		7,420	<=	10,000
Machine B	1,360	5,250	1,690	1,120		9,420	<=	12,000
Machine C	1,680	6,300	1,950	1,360		11,290	<=	15,000
Sales:								
Product 1	8					8	>=	8
Product 2		35				35	>=	20
Product 3			13			13	>=	12
Product 4				8		8	>=	8
Financing:								
Cash borrowings	48,000	175,000	52,000	40,000	**-165,000**	**150,000**	**=**	**800,000**
Debt covenant	-64,000	-245,000	-65,000	-48,000	572,000	**150,000**	**<=**	**150,000**