DEVELOPING NEW CAPABILITY INDICESFOR MEASURINGTHE POSITIONAL PERFORMANCEOFA MULTIDIMENSIONAL MACHINING PROCESS

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ABSTRACT

In this research, we propose three novel capability indices for measuring the positional performance of a multidimensional machining process under the assumption that the variances of machining results on different directions may not be equal. The statistical properties of the point estimators for the new capability indices are derived and their confidence intervals are established too. The numerical example shows that our proposed capability indices outperform the previous ones since they can better reflect the actual non-conforming rate of a multidimensional machining process.

Keywords：process capability analysis；engineering tolerance；positional tolerance；geometric dimensioning and tolerancing

Introduction

Geometric dimensioning and tolerancing (GD&T) is an engineering standard (ANSI Y14.5M-1994) providing a unified terminology and methodology for describing both the geometry of product features and their associated tolerances. Following these principles, the GD&T tolerance zone for the location of a hole is a circle circumscribing the square tolerance zone (i.e. positional tolerances). However, when positional tolerances are specified, the traditional process capability indices seem to be inadequate for measuring positional performance. In order to measure positional performance for a multidimensional machining process, Krishnamoorthi[8] proposed PC_p and PC_{pk} indices that are extensions of the C_p and C_{pk} indices. Assuming the process mean is equal to the target, Davis et al. [4] showed that the non-conforming rates for two-dimensional and three-dimensional cases are

$$\exp\left(-\frac{(U/\sigma)^2}{2}\right)$$

and

$$1-\left\{\Phi\left(\frac{U/\sigma}{\sqrt{2}}\right)-\frac{U}{\sigma}\sqrt{\frac{2}{\pi}}\exp\left(-\frac{(U/\sigma)^2}{2}\right)\right\}$$

, respectively and they proposed an index $R=U/\sigma$, where U is the radius of specification and σ is the standard deviation of quality characteristic. To measure the positional performance for a multidimensional machining process, Karl et al. [7] extended the concept of the multivariate process capability proposed by Taam et al. [10]. Moreover, Bothe [1] considered the radial distance between the target and the actual hole location as a quality characteristic to assess the capability of a process by locating the hole centers with in a circular tolerance zone.

Presently, the process capability indices for measuring the positional performance of a two or three dimensional machining process are developed under the assumption that the variances of machining results on different directions are equal. However, this assumption may not be true in the practical cases. For instance, Jackson [5] gave a practical example of a two-dimensional machining process, in which the variances of machining results on different directions (i.e. X or Y axis) are unequal. Moreover, due to the fact that the modern nano-cutting process can be considered as a special case of the multidimensional machining process, it is necessary to develop new capability indices for measuring the positional performance of both nano-cutting and traditional multidimensional machining processes under the unequal variances assumption. To provide quality practitioners a correct tool for measuring and evaluating the positional performance of a multidimensional machining process, not only the statistical properties for the point estimators of the new capability indices are derived, but also their confidence intervals are established. Moreover, the usefulness of our proposed indices is demonstrated through a simulation study and a numerical example.

Development of process capability index for spherical tolerance

Assuming a multidimensional machining process follows a multivariate normal distribution, then the tolerance region for a manufacturing process with the spherical tolerance can be written as:

$$(X_1 - t_1)^2 + \dots + (X_p - t_p)^2 \leq U^2, \tag{1.}$$

where $(X_1, X_2, \dots, X_p)$ is the actual location of machining results, U is the radius of specification and $(t_1, \dots, t_p)$ is the target location. Suppose that $(X_1, X_2, \dots, X_p)$ are independent, then the expected value of square of distance between the actual location and the target location is given by

$$E((X_1 - t_1)^2 + \dots + (X_p - t_p)^2) = \sum_{i=1}^{p}(\mu_i - t_i)^2 + \sum_{i=1}^{p}\sigma_i^2 \tag{2.}$$

,where μ_i and σ_i are the process mean and standard deviation of the *i*th quality characteristic, respectively. In order to properly evaluate the performance of process accuracy, we propose the following process capability index:

$$NPC_a = \frac{\sum_{i=1}^{p}(\mu_i - t_i)^2}{U^2}$$

(3.)

Since the value of the NPC_a index is large (small) as the distance between the process mean and the target location is large (small), the index NPC_a can provide information concerning process accuracy.

By Equations (1) and (2), a process precision index can be defined as:

$$NPC_p = \frac{U^2}{c_p \sum_{i=1}^{p} \sigma_i^2} \quad (4.)$$

,where $c_p = (\chi^2_{p,0.9973})^{p/2}/p$ and $\chi^2_{p,0.9973}$ is the α percentile of chi-square distribution with p degrees of freedom. Note that $c_1 = 2.9997$, $c_2 = 5.9145$ and $c_3 = 17.7542$. To evaluate both process precision and accuracy, we further define a process capability index as:

$$NPC_{pk} = NPC_p \times (1 - NPC_a) = \frac{U^2 - \sum_{i=1}^{p}(\mu_i - t_i)^2}{c_p \sum_{i=1}^{p} \sigma_i^2} \quad (5.)$$

Assuming a manufacturing process follows a multivariate normal distribution and let $\mathbf{X}_1, \ldots, \mathbf{X}_n$ be a random sample of n measurements with p quality characteristics from a multivariate normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Then, in order to estimate the NPC_a index, its estimator is given by

$$\widehat{NPC_a} = \frac{\sum_{i=1}^{p}(\bar{X}_i - t_i)^2}{U^2} \quad (6.)$$

,where $\bar{X}_i$ is the sample mean of quality characteristic X_i. Under the assumption of normality, the sampling distribution of $\widehat{NPC_a}$ can be written as:

$$\sum_{i=1}^{p} \frac{\sigma_i^2}{nU^2} \chi_1^2(\lambda_i)$$

(7.)

,where $\chi_1^2(\lambda_i)$ is a non-central chi-square distribution with 1 degree of freedom and non-central parameter $\lambda_i = (\mu_i - t_i)$. Moreover, the expected value and variance of $\widehat{NPC_a}$ can be derived as follows:

$$E(\widehat{NPC_a}) = NPC_a + \frac{\sum_{i=1}^{p} \sigma_i^2}{nU^2} \quad (8.)$$

$$Var(\widehat{NPC_a}) = \frac{2\sum_{i=1}^{p} \sigma_i^4}{n^2 U^4} + \frac{2\sum_{i=1}^{p} \sigma_i^2 (\mu_i - t_i)^2}{U^4} \quad (9.)$$

Note that $\widehat{NPC_a}$ is an asymptotically unbiased estimator of the NPC_a index. Similarly, the estimator of NPC_p index can be written as:

$$\widehat{NPC_p} = \frac{U^2}{c_p \sum_{i=1}^{p} s_i^2}$$

(10.)

,where $s_i^2 = \sum_{j=1}^{n}(X_j - \bar{X_i}^2)/(n-1)$ is the sample variance of the *i*th quality characteristic. Due to the complexity of the sampling distribution of $\widehat{NPC_p}$, it is approximated by

$$\frac{U^2 v}{\chi_f^2}, \quad (11.)$$

where χ_f^2 is a chi-square distribution with f degrees of freedom, $f = ((n-1)(\sum_{i=1}^{p} \sigma_i^2)^2)/(\sum_{i=1}^{p} \sigma_i^4)$ and $v = 2(\sum_{i=1}^{p} \sigma_i^4)/(n-1)$ (Patnaik, [9]). Moreover, the expected value and variance of $\widehat{NPC_p}$ index can be derived as follows:

$$E(\widehat{NPC_p}) = NPC_p \frac{f}{f-2} \quad (12.)$$

$$Var(\widehat{NPC_p}) = (NPC_p)^2 \frac{2}{(1-2/f)(f-4)}$$

(13.)

Apparently, $\widehat{NPC_p}$ is an asymptotically unbiased estimator of the NPC_p index. Based on the approximate sampling distribution of $\widehat{NPC_a}$, we further prove that an approximate 100(1-α)% confidence interval for the NPC_a index is

$$\left[\widehat{NPC_a} - Z_{\alpha/2}\sqrt{\frac{4\sum_{i=1}^{p} s_i^2(\bar{X}_i - t_i)^2}{nU^4}}, \widehat{NPC_a} + Z_{\alpha/2}\sqrt{\frac{4\sum_{i=1}^{p} s_i^2(\bar{X}_i - t_i)^2}{nU^4}}\right] \quad (14.)$$

,where Z_α is the α percentile of standard normal distribution. Furthermore, an approximate 100(1-α)% confidence interval for the NPC_p index can be written as

$$\left[\widehat{NPC}_p \frac{\chi^2_{\hat{f},1-\alpha/2}}{\hat{f}},\ \widehat{NPC}_p \frac{\chi^2_{\hat{f},\alpha/2}}{\hat{f}}\right] \quad (15.)$$

,where $\hat{f} = ((n-1)(\sum_{i=1}^{p} s_i^2)^2)/(\sum_{i=1}^{p} s_i^4)$ and $\chi^2_{f,\alpha}$ is the α percentile of a chi-square distribution with f degrees of freedom .

Given that a 99% confidence interval for the 0.95 coverage probability is equal to $0.95 \pm Z_{0.005}\sqrt{0.95 \times 0.05/5000} = 0.95 \pm 0.00794$, then we are 99% confident that the “true 95% confidence limit” ranges from 0.94206 to 0.95794. Based on the simulation results, we find that the observed 95% coverage probabilities for NPC_a and NPC_p indices are within the nominal interval at 99% confidence level when sample size is large. Hence, the accuracy of the confidence interval in Equations (14) and (15) can be ascertained.

Numerical example

Jackson [5] gave a practical example of a two dimensional machining process, in which the quality characteristic is the location (X, Y) of a circular hole. The radius of specification is $U = 0.18$ and the target location $(t_1, t_2) = (-8.37, 137.5)$. After performing the Kolmogorov-Smirnov test, we found that the 300 collected measurements follow a multivariate normal distribution with the sample mean $\bar{\mathbf{X}}' = [-8.25, 137.56]$ and the sample covariance matrix $\mathbf{S}$, where

$$\mathbf{S} = \begin{bmatrix} 0.00621 & -0.00024 \\ -0.00024 & 0.00342 \end{bmatrix}.$$

Both Pearson-Correlation and Levene tests are then performed to test assumptions of independence and equal variances. The test results indicate that the characteristics are independent and the variances are unequal since the p-values are 0.3616 and 0.001, respectively. The scatter plot of holes location for a two dimensional machining process is shown in Figure 1.

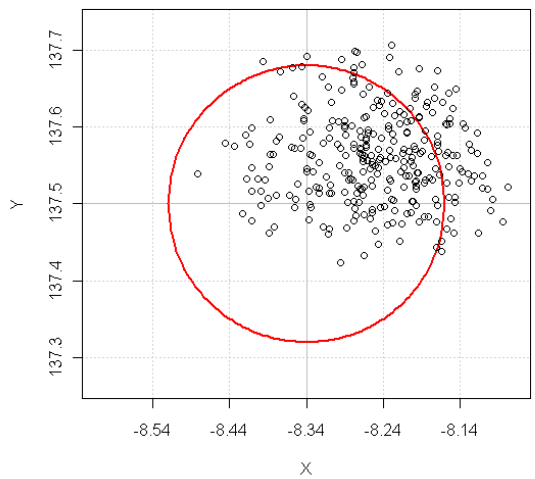

FIGURE 1 Scatter plot of holes location for a two dimensional machining process

As can be seem from the scatter plot, the process mean is deviated from the target location and the process variance is large with respect to the specification. The new capability indices and their associated interval estimates are summarized in Table 1. Since $NPC_a = 0.354$and $NPC_p = 0.570$, one can conclude that the process exhibits high manufacturing risk since it lacks of process accuracy and precision. Moreover, the information of process accuracy and precision revealed by our proposed indices, NPC_a, NPC_p and NPC_{pk} can lead to a clear direction for future quality improvement. Due to the complexity of the sampling distribution of estimator of the NPC_{pk} index, we utilize a bootstrapping approach suggested by Chou et al. [3] to obtain the biased-corrected and accelerated confidence interval for the NPC_{pk} index.

TABLE 1.The new capability indices and their associated interval estimates for a two dimensional machining process

	New Capability Indices		
	NPC_a	NPC_p	NPC_{pk}
Index values	0.354	0.570	0.368
Interval Estimates	[0.299, 0.409]	[0.482, 0.664]	[0.323, 0.416]

Conclusion

The process capability index is commonly used in industry to measure and evaluate the process performance. Several capability indices have been developed for measuring the positional performance of a multidimensional machining process under the assumption that the variances of machining results on different directions are equal. However, this assumption may not be true in most practical situations. In this research, we propose three novel capability indices, NPC_a, NPC_p and NPC_{pk} for measuring the precision and accuracy of the positional performance of a multidimensional machining process under the assumption that variances of machining results on different directions may not be equal. Through a simulation study, we have demonstrated that the actual positional performance can be accurately reflected by using our proposed indices and the R index proposed by Davis et al. [4]. Comparing with the R index, our new capability indices have the advantage of providing information for process precision and accuracy. Moreover, the statistical properties of the point estimators for the new capability indices and their associated confidence intervals are derived. These statistical properties may lead to sample size determination and can be served as a useful reference for quality practitioners. Finally, the simulation results and the numerical example show that our proposed capability indices outperform previous indices since they can better reflect the actual non-conforming rate of a multidimensional machining process.

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