

OLIGOPOLY WITH BOUNDED OUTPUT ADJUSTMENTS

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ABSTRACT

An n -firm classical oligopoly is considered first with linear price and cost functions which have a unique equilibrium. We assume the output adjustments of the firms are bounded in a sense that they are not willing to make small changes, and the output adjustments are bounded from above, which are realistic assumptions in real economies. The best responses of the firms are first determined and the existence of infinitely many equilibria is verified. Similar conclusions can be obtained with hyperbolic price function as well.

INTRODUCTION

Assume n firms produce the same product or offer identical service to a homogeneous market. Let x_k denote the output of firm k , $s = \sum_{k=1}^n x_k$ the output of the industry and $s_k = \sum_{l \neq k} x_l$ the output of the rest of the industry from firm k 's perspective. If $p(s)$ is the unit price function and $C_k(x_k)$ is the production cost of firm k , then its profit is given as

$$\Psi_k = x_k p(s) - C_k(x_k) \quad (1)$$

In this way an n -person game is defined, where the firms are the players, the set of non-negative real numbers is the strategy set of each player, and the payoff of player k is given by equation (1). This game is one of the most frequently studied models in mathematical economics, which has a huge literature. A comprehensive summary of the earliest results is given in Okuguchi [2], their multiproduct generalization with case studies are discussed in Okuguchi and Szidarovszky [3]. The studied models were mainly linear making equilibrium and stability analysis relatively simple. The attention recently turned to the analysis of nonlinear models. Bischi et al. [1] offers a review of the most recent development in this area.

In this paper linear price and cost functions are assumed first, in which case there is a unique equilibrium. We introduce the additional assumption that the firms are not willing to make very small output adjustments, since the small profit increase is insufficient incentive to perform all required adjustments in the production process. Because of limited resources it is also assume that the amount of output increase in each time period is bounded from above. These additional assumptions result in nonlinear, discontinuous best response functions and infinitely many steady states of the dynamic system. Similar conclusions can be reached given a hyperbolic price function.

MATHMETICAL MODELS

Assume linear price and cost function: $p(s) = A - Bs$ and $C_k(x_k) = c_k x_k + d_k$ ($1 < k < n$). The profit of firm k is quadratic,

$$\Psi_k = x_k(A - Bx_k - Bs_k) - (c_k x_k + d_k) \quad (2)$$

which is a concave parabola in x_k . The stationary point x_k is derived by simple differentiation,

$$\frac{\partial \Psi_k}{\partial x_k} = A - 2Bx_k - Bs_k - c_k = 0$$

implying that

$$x_k^* = \frac{A - c_k - Bs_k}{2B} \quad (3)$$

We can assume that $x_k^* > 0$, otherwise the optimal production level of firm k is zero, the firm leaves the market. x_k^* can be considered as the best response of firm k ,

$$R_k \left(\sum_{l \neq k} x_l \right) = \frac{A - c_k - B \sum_{l \neq k} x_l}{2B} \quad (4)$$

The dynamic extension of this model with discrete time scales and adjustments toward best responses is modeled by the system of difference equations

$$x_k(t+1) = x_k(t) + K_k \left[R_k \left(\sum_{l \neq k} x_l(t) \right) - x_k(t) \right] \quad (5)$$

for $k = 1, 2, \dots, n$, where K_k is the speed of adjustment of firm k .

Assume next that the firms do not want to make adjustment below a certain threshold ε_k and cannot make adjustment larger than a specified positive threshold Δ_k . Based on these additional conditions the modified best response of firm k becomes

$$R_k \left(\sum_{l \neq k} x_l, x_k \right) = \begin{cases} x_k & \text{if } |x_k^* - x_k| \leq \varepsilon_k \\ x_k + \Delta_k & \text{if } x_k^* \geq x_k + \Delta_k \\ x_k^* & \text{otherwise} \end{cases} \quad (6)$$

Figure 1 illustrates this function. It is nonincreasing in s_k , however it is discontinuous and nonlinear.

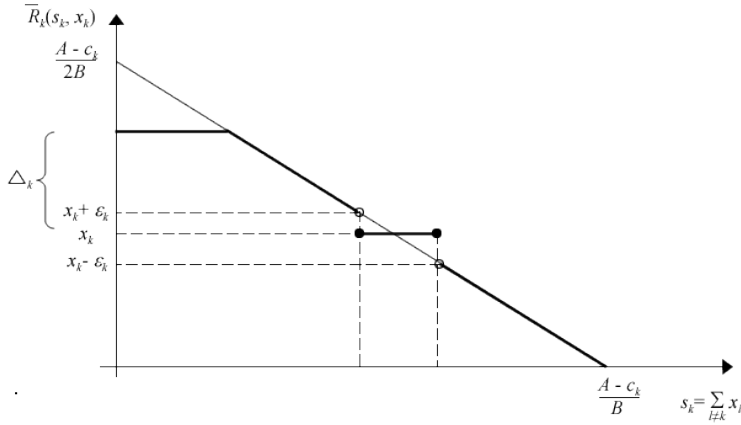


Figure 1. Best response of firm k , linear price

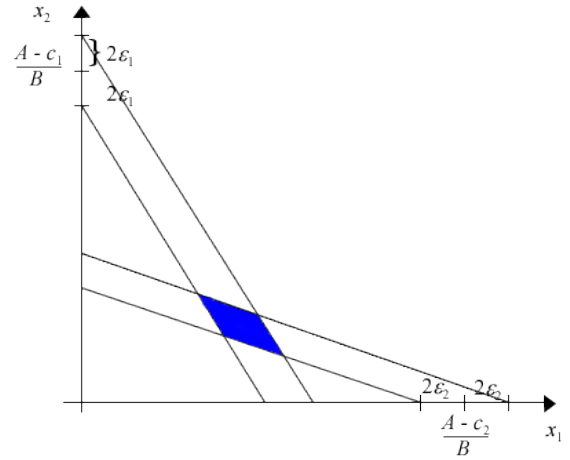


Figure 2. Set of steady states for $n = 2$, linear price

With the additional assumptions the linearity of the model is lost. The corresponding dynamic model has now the form

$$x_k(t+1) = x_k(t) + K_k \left[\bar{R}_k \left(\sum_{l \neq k} x_l(t), x_k(t) \right) - x_k(t) \right] \quad (k = 1, 2, \dots, n) \quad (7)$$

Notice that the right hand side is discontinuous and nonlinear.

Steady State Analysis

Consider first model (5), in which case an output vector $(\bar{x}_1, \dots, \bar{x}_n)$ is a steady state if and only if for all k ,

$$\frac{A - c_k - B \sum_{l \neq k} \bar{x}_l}{2B} = \bar{x}_k,$$

Implying that

$$\bar{x}_k = \frac{A - c_k - B \bar{s}}{B} \quad (8)$$

By adding this equation for $k = 1, 2, \dots, n$ we have

$$\bar{s} = \frac{nA - \sum_{l=1}^n c_l - nB\bar{s}}{B}$$

that is,

$$\bar{s} = \frac{nA - \sum_{l=1}^n c_l}{(n+1)B} \quad (9)$$

Then from (8) we can derive the equilibrium output levels of the firms:

$$\bar{x}_k = \frac{A + \sum_{l=1}^n c_l - (n+1)c_k}{(n+1)B} \quad (10)$$

The steady state analysis of model (7) is much more complicated. An output vector $(\bar{x}_1, \dots, \bar{x}_n)$ is a steady state of system (7) if and only if for all k ,

$$\left| \frac{A - c_k - B \sum_{l \neq k} \bar{x}_l}{2B} - \bar{x}_k \right| \leq \varepsilon_k \quad (11)$$

which can be rewritten as

$$\frac{A - c_k}{B} - 2\varepsilon_k \leq \sum_{l \neq k} \bar{x}_l + 2\bar{x}_k \leq \frac{A - c_k}{B} + 2\varepsilon_k \quad (12)$$

Notice that the steady state (10) clearly satisfies these relations and since $\varepsilon_k > 0$ for all k , inequalities (12) have infinitely many solutions. Thus system (7) has infinitely many steady states, and the steady state set is a bounded polyhedron.

Example. Assume $n = 2$, then relations (12) can be rewritten as

$$\frac{A - c_1}{B} - 2\varepsilon_1 \leq x_2 + 2x_1 \leq \frac{A - c_1}{B} + 2\varepsilon_1 \quad (13)$$

and

$$\frac{A - c_2}{B} - 2\varepsilon_2 \leq x_1 + 2x_2 \leq \frac{A - c_2}{B} + 2\varepsilon_2 \quad (14)$$

Figure 2 illustrates the set of feasible solutions.

CASE OF HYPERBOLIC PRICE FUNCTION

Assume now that the price function is hyperbolic, $p(s) = A/s$, which is a common assumption in the literature (Bischi et al., 2010). If the cost function is linear, then the profit function of firm k has the form

$$\Psi_k = \frac{Ax_k}{x_k + s_k} - (c_k x_k + d_k) \quad (15)$$

Similarly to the linear case, the stationary point is given as

$$x_k^* = \sqrt{\frac{As_k}{c_k}} - s_k \quad (16)$$

and the best response of firm k with the additional assumptions is also given by relation (6) with the only difference that x_k^* has now a different form. This best response function is illustrated in Figure 3.

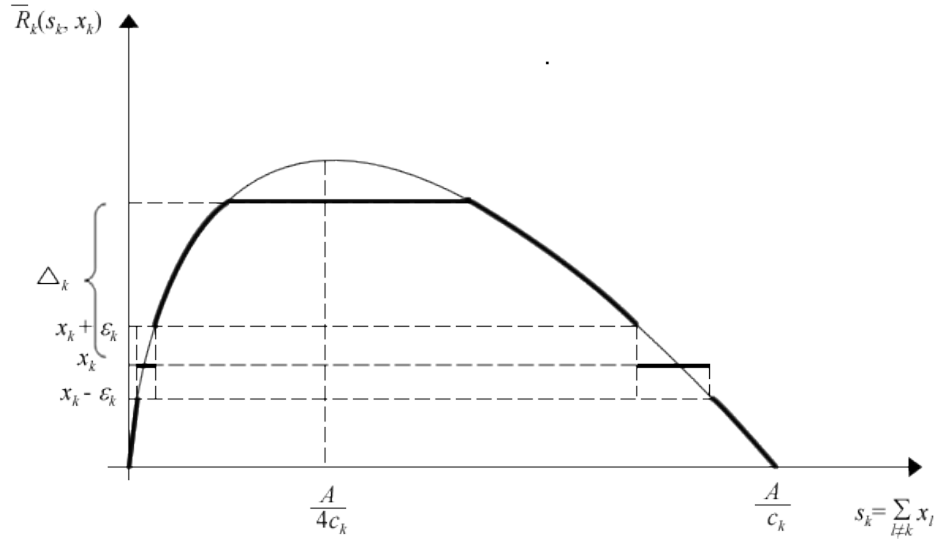


Figure 3. Best response of firm k , hyperbolic price

The set of steady states is now a nonlinear set, and for $n = 2$ it is shown in Figure 4.

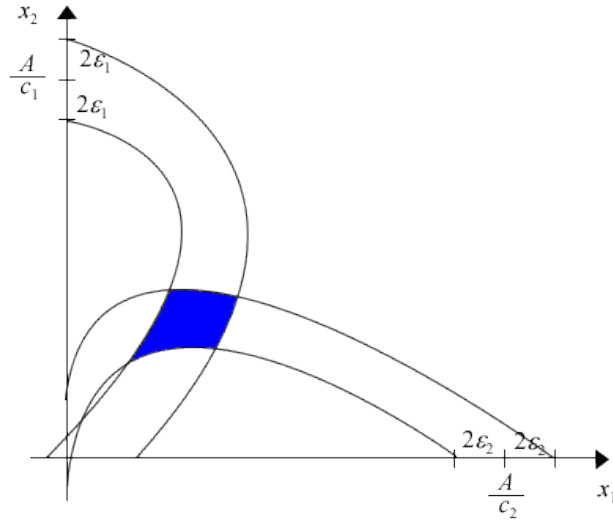


Figure 4. Set of steady states for $n = 2$, hyperbolic price

CONCLUSION

In this paper we introduced two realistic assumptions into the well-known oligopoly model. If the price and all cost functions are linear, then the additional assumptions result in the loss of the linearity of the dynamic model and the uniqueness of the steady state. Even the model is non-linear, the set of all steady states is a linear set. Similar conclusion could be reached with hyperbolic price function as well.

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