APPLICATION OF THE NELSON SIEGEL YIELD CURVE MODEL IN AN ILLIQUID AND UNDEVELOPED FINANCIAL MARKET

Davor Zoricic, Faculty of Economics and Business, University of Zagreb, Trg J.F. Kennedyja 6, 10 000 Zagreb, Croatia, 00385-1-238-3103, dzoricic@efzg.hr
Silvije Orsag, Faculty of Economics and Business, University of Zagreb, Trg J.F. Kennedyja 6, 10 000 Zagreb, Croatia, 00385-1-238-3109, sorsag@efzg.hr

ABSTRACT

This paper examines the possibility of applying the Nelson Siegel yield curve model in the Croatian financial market. In such an illiquid and undeveloped financial market yield curve modeling presents a special challenge primarily regarding the available market data. The use of the model is limited compared to the developed markets and the interpretation of the resulting yield curve requires much more cautiousness. However this paper clearly shows that the yield curve model is able to capture changes in the business cycle according to the macroeconomic theory and therefore provide valuable information to the financial industry and other economic subjects.

INTRODUCTION

The yield curve plays a crucial role in the modern financial markets. A wide body of literature has therefore appeared since 1970’s concerning the yield curve modeling. Most of the research (especially during the 20th century) has reasonably been more or less directly related to the most developed financial markets in the world. More recently a significant portion of research dedicated to yield curve modeling in the illiquid and undeveloped financial markets has emerged wherever such conditions in the financial markets may appear. In the past decade papers have been published regarding yield curve modeling in the financial markets of India, Taiwan, Russia, and Serbia among others. Liquidity issues have been specifically addressed in the work of Dutta et al. (2005), Chou (2009) and Smirnov and Zakharov (2003) concerning the Indian, Taiwanese and Russian financial markets respectively [3] [1] [8]. In the mentioned papers various yield curve modeling approaches were considered and empirically tested including the parametric Nelson Siegel model. But papers referring to the probably the least developed financial market - Serbian financial market continuously used the Nelson Siegel model exclusively. Papers published by the Jefferson Institute (2005), Drenovak (2006) and Zdravkovic (2010) modeled the yield curve relying only on the Nelson Siegel model [5] [2] [9]. The popularity of the model can mostly be attributed to its relative simplicity which does not affect the model’s ability to fit the market data reasonably well.

METHODOLOGY

Nelson and Siegel (1987.) have proposed a parsimonious yield curve model that is flexible enough to produce upward sloping, downward sloping and humped yield curves [6]. Although it rests only on estimation of four parameters the model is able to capture the most common shapes the yield curve takes on in practice. In their research the authors approach the yield curve modeling by defining the forward yield curve in order to ensure that the resulting forward curve is smooth and asymptotic which are desirable properties from theoretical standpoint [4, p. 62]. They present the following equation [6, p. 475]:

$$f(n,\beta) = \beta_1 + \beta_2 e^{\left(\frac{-n}{\lambda}\right)} + \beta_3 \frac{n}{\lambda} e^{\left(\frac{-n}{\lambda}\right)} \tag{1}$$

where the $f(n,\beta)$ represents the forward yield curve function which depends on maturity *n* and parameters λ, $ß_1$, $ß_2$ *and* $ß_3$. Parameter $ß_1$ represents asymptote while parameters $ß_2$ *and* $ß_3$ enable estimation of various shapes of the yield curve allowed by the model. If equation (1) is integrated and divided by *n* a zero-coupon yield curve can be derived (zero-coupon yield is an average of the forward yields). Furthermore if the parameters $ß_1$, $ß_2$ *and* $ß_3$ are grouped we get [9, p. 28]:

$$\mathrm{Z_n} = \beta_1 + \beta_2\left(\frac{1-\mathrm{e}^{-\lambda \mathrm{n}}}{\lambda \mathrm{n}}\right) + \beta_3\left(\frac{1-\mathrm{e}^{-\lambda \mathrm{n}}}{\lambda \mathrm{n}} - \mathrm{e}^{-\lambda \mathrm{n}}\right) \tag{2}$$

where $\mathrm{Z_n}$ represents the zero-coupon yield curve function and *ß* parameters represent yield curve factors. Thus $ß_1$ refers to the level, $ß_2$ to the slope and $ß_3$ to the curvature of the yield curve. Expressions in parenthesis are called factor loadings which are defined by maturity *n* and factor λ.

Equation (2) can be written as a regression equation taking on the functional form given by equation (3):

$$y_t(\tau) = H * \beta_t + e_t, \tag{3}$$

where $y_t(\tau)$ represents a vector of yields observed in the moment *t* for *T* maturities in the vector τ, error term is given by $e \sim N(0,R)$ and H represents factor loadings matrix [7, p. 71]:

$$H = \begin{bmatrix} 1 & \frac{1-\exp(-\lambda\tau_1)}{\lambda\tau_1} & \frac{1-\exp(-\lambda\tau_1)}{\lambda\tau_1} - \exp(-\lambda\tau_1) \\ 1 & \frac{1-\exp(-\lambda\tau_2)}{\lambda\tau_2} & \frac{1-\exp(-\lambda\tau_2)}{\lambda\tau_2} - \exp(-\lambda\tau_2) \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-\exp(-\lambda\tau_T)}{\lambda\tau_T} & \frac{1-\exp(-\lambda\tau_T)}{\lambda\tau_T} - \exp(-\lambda\tau_T) \end{bmatrix}. \tag{4}$$

Also following the work of Nyholm (2008.) it can be noted that above stated expression can be linearized if the parameter λ is treated as a constant rather than being a variable that needs to be estimated. In such a scenario the regression equation given by (3) is solved for different values of parameter λ chosen arbitrarily from a predetermined interval. The optimal value for parameter λ is then finally determined as the one which minimizes the sum of squared residuals between the observed and yields estimated by the model [7, p. 71-72]. The described approach has been adopted and pursuit in this paper.

Once estimated, the parameters of the Nelson Siegel model can be used to estimate the whole term structure (i.e. estimate any yield for any given maturity) for any given point in time in the analyzed sample.

DATA

Since the Croatian government has only issued 15 bonds in the past 20 years yield curve was estimated using both data on government bonds and treasury bills. The available instruments are sometimes issued as pure kuna instruments and sometimes as instruments with foreign currency (euro) clause. Therefore two separate samples were initially formed regarding this distinctive characteristic. Mid yields were collected via Bloomberg on all government bonds. For treasury bills Ministry of finance data was used which refers to the yields achieved at regular auctions. Since there is little trading activity when it comes to treasury bills this was the best source of data possible. Monthly averages were calculated for all the instruments based on collected yields which enabled yield curve estimation on monthly basis.

After looking at the collected data for two samples it became clear that there were a lot of months with no trading activity or worse with no instruments to be traded which resulted in only one or two data points available per month. To be exact for sample referring to the pure kuna instruments 4 or more data points have been available continuously only starting from April 2006. For the foreign currency clause sample 4 or more data points have been available from March 2004. Market data have been collected for both samples up to June 2011. Thus samples range from April 2006 to June 2011 (63 observations) for the pure kuna sample and from March 2004 to June 2011 (88 observations) for the foreign currency clause sample.

As financial markets convention is to report yields on government bonds on yield to maturity basis, zero-coupon yields had to be calculated using the bootstrapping technique for both samples. This was carried out in Matlab software using Matlab's "zbtyield" function.

RESEARCH RESULTS

Parameters of the Nelson Siegel model were estimated for both samples of the collected data by using a Matlab code which carried out calculations specified by the equation (3). Estimated parameters were then used to estimate the term structure (the yield curve) for every observation in both data samples. For each observation yields were estimated for the following maturities: 3, 6, 9, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months.

Two problems appeared in the process. First, it turned out that standard errors of the parameter estimators were approaching infinity for some observations. This appears to be the case in both data samples whenever the Nelson Siegel model parameters are estimated based on only 4 data points for given observation. Therefore it seems that at least 5 data points are necessary to estimate the model parameters properly. Due to the described problem the sample referring to the pure kuna instruments was reduced by 1 observation to 62 observations, while the sample referring to the foreign currency clause instruments had to be reduced by 11 observations to 77 observations. Second, in some instances estimated model parameters resulted in unusually low or high yield estimates at the short end of the yield curve (maturities up to 1 year but mostly regarding maturities referring to 3 and 6 months). This kind of a problem also appeared in both data samples whenever an observation lacked data on the short end of the yield curve. Obviously when this is the case the model adjusts itself too much to the available data leading to distortion in the estimates of the short end of the curve. As this problem was also more evident in the foreign currency clause instruments sample graph in Figure 2 was adjusted to show only maturity spectrum ranging from 2 to 10 years.

Regardless of the encountered problems importance of the estimated yield curves is at least twofold. Firstly, yield curves for both samples seem to exhibit changes in their shapes in line with the macroeconomic theory (inverted at the beginning of the world financial crisis, steeply upward sloping in the lasting recession, mildly upward sloping during economic expansion, etc. – see Figure 1 and 2) which makes a yield curve model a desirable tool when trying to form a clear picture regarding the yield curve shape and its dynamics. Financial market's illiquidity does not seem to diminish or distort this significantly. Secondly, unobservable market yields can be extrapolated and interpolated when using a model. In undeveloped financial markets this is particularly useful as it enables continuous availability of arbitrarily chosen maturities.

The application of such a yield curve model in such an environment however lacks the potential to serve in the field of risk management and derivatives but allows valuable insight regarding market expectations. The latter could even be used to some extent when it comes to bond valuation and pricing.

CONCLUSION

The paper deals with modeling the yield curve in an illiquid and undeveloped financial market. It shows that it is possible to use the Nelson Siegel yield curve model in such an environment to model the yield curve in spite of the challenges concerning primarily the available market data.

The research results seem to suggest that a minimum of 5 data points need to be available for every observation in the sample in order to estimate the Nelson Siegel yield curve model parameters properly. Also data should cover as much of the maturity spectrum as possible, especially at the short end of the curve to prevent distortions in the estimates caused by the lack of data.

Furthermore the estimated yield curves for the analyzed data samples seem to show that the yield curve evolution is in line with the macroeconomic theory over the analyzed period. Therefore it is possible to apply the Nelson Siegel model with regard to collecting information on market expectations. Neither the analyzed yield curve model nor the available data support the application of estimated yield curves in the field of risk management.

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FIGURE 1

Yield curve evolution estimated by Nelson Siegel model (pure kuna instruments)

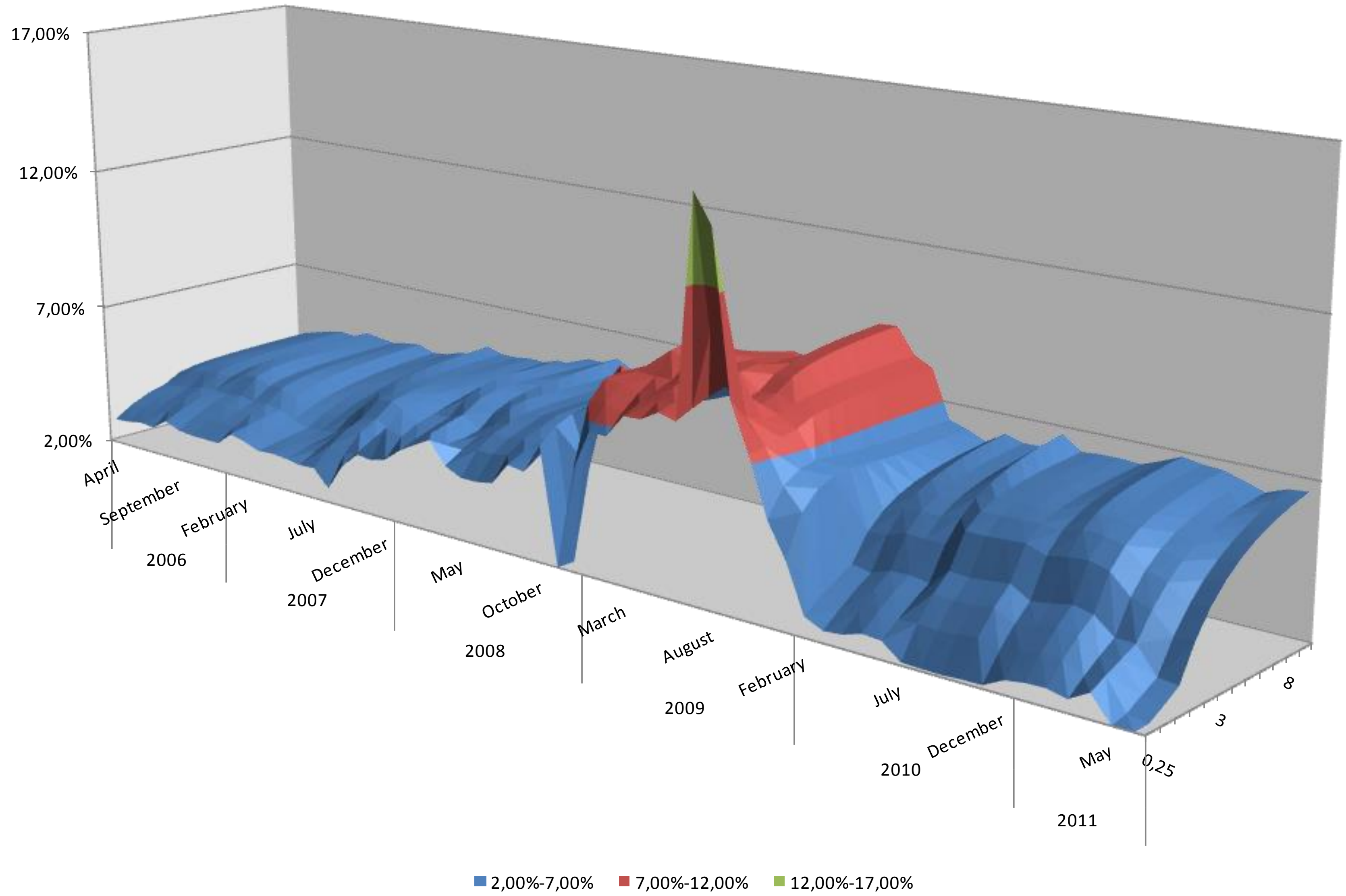

FIGURE 2
Yield curve evolution estimated by Nelson Siegel model (foreign currency clause instruments)