PROJECT SELECTION WITH SOCIAL CHOICE METHODS

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ABSTRACT

A project selection problem is discussed when the panel members have only rankings of the decision alternatives without quantifiable utility functions. Social choice methods are introduced and applied: plurality voting, Borda counts, Hare systems, pair-wise comparisons and dictatorship. These methods are used with both equal and non-equal panel members.

1. INTRODUCTION

It is very common decision making scenario when a group of people has to reach a common outcome. In the decision theory, there are several different concepts modeling and solving such problems.

The members can be considered as the players in an *n*-person game and the payoff of each player has an unfavorable value in the absence of an agreement, and in an agreement each player receives the corresponding payoff. The game can be modeled as a noncooperative or a cooperative game obtaining the Nash-equilibrium or any cooperative solution (Forgo et al., 1999). If the members of the group have conflicting priorities, then conflict resolution methodology can be used including the Nash bargaining solution, the area monotonic solution among others. The problem also can be modeled as a multiobjective optimization problem. This is especially the case when a mediator is hired and he is playing the role of a single decision maker. In the literature there are many different solution concepts and methods to select from (Szidarovszky et al., 1986). All these methods assume that each member of the group has a quantifiable utility function what he wants to maximize. There are however decision problems when no such utility function is available, the members of the group can only give rankings of the decision alternatives. The data can be arranged in an $M \times (N+1)$ matrix, where the rows correspond to the members of the group, the first N columns contain the rankings of the members and the last column gives the relative powers of the members. Since only rankings are available for the alternative, the first Nelements of each row is a permutation of the integers $1, 2, \ldots, N$, where 1 is given to the most preferred alternative, and so on, $N$is given to the least preferred one. Table 1 gives the data for a project selection decision when 5 members of a panel have to select one of given 4 proposals to be supported. The members are not considered equal, higher weight is given to a member if he is often a participant of such panels, having larger experience and his opinion is more trustable. Member 4 is a frequent participant, so he has the highest weight. Members 1 and 2 are new comers, this is their first participation, so they get the lowest weight.

2. METHODOLOGY

Let a_{ij} denote the (i, j) element of the data matrix which shows the ranking of proposal j by member i. Its value is one of the integers $1, 2, \ldots, N$. In addition let w_i denote the weight of member i. There are several methods to find a commonly acceptable outcome (Taylor, 1995).

	P1	P2	P3	P4	
M1	2	1	3	4	1
M2	1	3	2	4	1
M3	4	1	3	2	2
M4	2	4	1	3	3
M5	4	3	2	1	2

Table 1. Data for project selection problem

In applying Plurality voting we have to determine the number of total weighted best rankings for each alternative and the outcome is the one with the largest number of weighted votes.

Mathematically this method can be described as follows. Define

$$\Delta_{ij} = \begin{cases} 1 & if\ a_{ij} = 1 \\ 0 & otherwise \end{cases} \quad (1)$$

for all *i* and *j*, and let

$$n_j = \sum_{i=1}^{N} w_i \Delta_{ij} \quad (2)$$

for each alternative *j*, which gives the total number of weighted votes. The outcome is then chosen as j_0 if

$$n_{j_0} = max\{n_1, n_2, \ldots, n_N\} \quad (3)$$

In our case

$n_1 = 1$, $n_2 = 3$, $n_3 = 3$ and $n_4 = 2$

So alternatives 2 and 3 are equally the best. So using this method we cannot distinguish between these alternatives. The application of another method is suggested in such cases. The disadvantage of this method is the consideration of only the best rankings and it is not sensitive to lower rankings.

The Border countstake all rankings into account by adding the weighted rankings by all members:

$$B_j = \sum_{i=1}^{N} w_i a_{ij} \quad (4)$$

and the alternative j_0 with the smallest count is the choice,

$$B_{j_0} = min\{B_1, B_2, \ldots, B_N\} \quad (5)$$

In the case of Table 1 we have

$B_1 = 25$, $B_2 = 24$, $B_3 = 18$, $B_4 = 23$,

so alternative 3 is the solution.

The Hare system is based on repeted deletions. If there is an alternative which has more than half of the weighted votes, then it is the solution and the procedure terminates. Otherwise delete the alternative with the least number of weighted votes, adjust the table accordingly and start again. If alternative j^* is deleted, then the a_{ij} values are adjusted as

$$a_{ij}^{new} = \begin{cases} a_{ij} & if\ a_{ij} < a_{ij^*} \\ a_{ij} - 1 & if\ a_{ij} > a_{ij^*} \end{cases} \tag{6}$$

In applying plurality voting we have already computed the total weighted number of votes, no alternative has more than 4.5 votes and alternative 1 has the smallest value. So it is eliminated from the table. The resulting table is shown in Table 2. The new total weighted numbers of votes are

$n_2 = 3$, $n_3 = 4$, $n_4 = 2$

	P2	P3	P4	
M1	1	2	3	1
M2	2	1	3	1
M3	1	3	2	2
M4	3	1	2	3
M5	3	2	1	2

Table 2. First reduced table

so alternative 4 has to be deleted. The reduced table is shown in Table 3, in which alternative 2 has 3 and alternative 3 has 6 votes, so alternative 3 is the final choice.

	P2	P3	
M1	1	2	1
M2	2	1	1
M3	1	2	2
M4	2	1	3
M5	2	1	2

Table 3. Second reduced table

In applying Pairwise comparisons we have to define a rule in comparing any pair of alternatives. Let $NW = \sum_{i=1}^{N} w_i$ and $N(j_1, j_2)$ = weighted number of members such that $a_{ij_1} > a_{ij_2}$ which shows how many members give higher ranking to alternative j_1 than to alternative j_2. We say that j_1 is overall better than j_2 if $N(j_1, j_2) > \frac{NW}{2}$. If $N(j_1, j_2) = \frac{NW}{2}$, then the two alternatives are considered equal. These relations are denoted as $j_1 > j_2$ and $j_1 \sim j_2$, respectively. In our case $NW = 9$ and

$N(1, 2) = 4 < 4.5$
$N(1, 3) = 2 < 4.5$
$N(1, 4) = 5 > 4.5$
$N(2, 3) = 3 < 4.5$
$N(2, 4) = 4 < 4.5$

$N(3, 4) = 5 > 4.5$

Notice that $N(j_2, j_1) = NW - N(j_1, j_2)$ so if $N(j_1, j_2) < \frac{NW}{2}$, then $j_2 > j_1$, if $N(j_1, j_2) > \frac{NW}{2}$ then $j_1 > j_2$ and if $N(j_1, j_2) = \frac{NW}{2}$ then $j_1 \sim j_2$. So we have

2>1, 3>1, 1>4, 3>2, 4>2 and 3>4.

These preferences are shown in a graph presented in Figure 1.

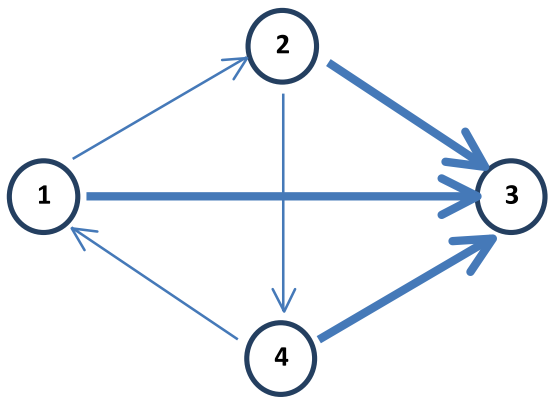

Figure 1. Preference graph

Clearly alternative 3 is the choice, since it is better than all other alternatives. There is however a problem with this graph. Consider alternatives 1, 2, and 4,

2>1, 1>4 and 4>2.

The transitivity property of preferences would require that2>1 and 1>4 imply 2>4, however we have here the opposite direction. In such cases the resulted circles are deleted from the graph so only the thick arcs should remain in Figure 1, which results in the same final decision.

In applying Dictatorship one member is selected as the dictator and his best choice should be selected by the group. Since member 4 has the largest weight, it is logical to choose him as the dictator, and then his best choice, alternative 2 is the decision of the group.

If the members of the group are equal, then $w_i = 1$ for all i. In applying Plurality voting we have

$n_1 = n_3 = n_4 = 1$ and $n_2 = 2$,

so alternative 2 is the choice.

The Border counts are as follows:

$B_1 = 13$, $B_2 = 12$, $B_3 = 11$, $B_4 = 14$, so alternative 3 is the decision.

In applying <u>Hare-systems</u> we have a problem, since alternatives 1, 3 and 4 have the lowest number of votes, so either one of them can be eliminated. Table 4 shows the three reduced tables. The number of votes are$n_2 = 2$, $n_3 = 2$, $n_4 = 1$; $n_1 = 2$, $n_2 = 2$, $n_3 = 1$; $n_1 = 1$, $n_2 = 2$, $n_3 = 2$ respectively so in the further reduction alternatives 4, 4 and 1 are deleted. The results after the second reduction are shown in Table 5.

	P2	P3	P4	P1	P2	P4	P1	P2	P3
M1	1	2	3	2	1	3	2	1	3
M2	2	1	3	1	2	3	1	3	2
M3	1	3	2	3	1	2	3	1	2
M4	3	1	2	1	3	2	2	3	1
M5	3	2	1	3	2	1	3	2	1

Table 4. Reduced tables with equal members

	P2	P3	P1	P2	P2	P3
M1	1	2	2	1	1	2
M2	2	1	1	2	2	1
M3	1	2	2	1	1	2
M4	2	1	1	2	2	1
M5	2	1	2	1	2	1

Table 5. Second reduced tables with equal members

In the first case P3 is the choice, since it got 3 votes. In the second case P2, and in the third case P3 is the choice.

In <u>Pair-wise comparisons</u> we get

$N(1, 2) = 2$
$N(1, 3) = 2$
$N(1, 4) = 3$
$N(2, 3) = 2$
$N(2, 4) = 3$
$N(3, 4) = 3$

So we conclude the following preferences:

2>1, 3>1, 1>4, 3>2, 2>4, 3>4

with the graph shown in Figure 2. Clearly alternative 3 is the choice.

<u>Dictatorship</u> has no sense since the members are equal.

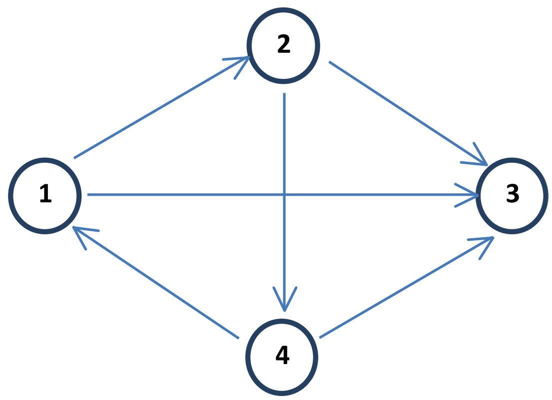

Figure 2. Preference graph with equal members

3. CONCLUSIONS

A project selection problem is examined when the panel members have no quantifiable utility functions, only rankings of the decision alternatives. Five simple methods were introduced and used with both equal and non-equal members: plurality voting, Border counts, Hare systems, pair-wise comparisons and dictatorship. The procedures are very simple, so the panel members can see and understand the selection procedure, so they can accept the results much easier than after applying a "black-box" type method.

4. REFERENCES

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