UNRAVELING THE PARADOX OF LEAD-TIME UNCERTAINTY

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ABSTRACT

Researchers have revealed that a reduction in lead-time uncertainty could paradoxically increase inventories. The scholarly consensus is that this paradox is important primarily for two reasons. First, the phenomenon comes into play at common in-stock service levels for most firms. Second, the paradox implies that conventional use of the normal distribution to characterize out-of-stock exposure risk will produce flawed prescriptions for managing lead time—namely, that less uncertainty would benefit safety inventory. This research note unravels the paradox of lead-time uncertainty and assesses its importance to supply chain practitioners.

INTRODUCTION

Scholars have discovered that less lead-time uncertainty may paradoxically increase inventory. More importantly, the scholarly consensus is that this paradox comes into play for common service levels, where it would invalidate prescriptions for managing lead-time based on the normal approximation of the distribution of demand over the out-of-stock (OOS) exposure period. This research note unravels the paradox of lead-time uncertainty (hereafter, the paradox) and assesses its importance to effective lead-time management. The research questions are as follows:

1. What are the conditions that enable the paradox to come into play at common service levels?
2. What is the impact of lead-time uncertainty on inventory under these conditions?

BACKGROUND AND LITERATURE REVIEW

Song [13] revealed that it is possible for safety inventory to increase in response to a decrease in lead-time variability in a simple base-stock system. Song et al. [14] proved that this paradoxical phenomenon could also occur in stochastic continuous-review systems. Meanwhile, Chopra et al. [3] argued that the phenomenon affects most firms and demonstrated how the normal approximation of OOS exposure risk would produce flawed prescriptions for managing lead-time levers. For some scholars, these demonstrations provided additional evidence of the fallacy of using the normal distribution [2], [8], [16]. The study encouraged others to consider different candidate distributions, [1], [6], [9], [10], [12]. Additionally, it inspired Dullaert and Zamparini [5] to use paradox as a novel explanation for inconsistent and sometimes contradictory shippers' perceptions of the value of less lead-time uncertainty.

Perhaps, the best way to understand the nature of the paradox is to compare the effects of a decrease in lead-time variability on both symmetrical and asymmetrical statistical forms under identical conditions. For example, assume that the distribution of demand (D) has a mean of 20 units/period and a standard deviation of 15 units/period, while the distribution of lead-time (L) has a mean of 10 periods and a standard deviation of 7 periods. If D and L are independent random variables and X represents demand over the OOS exposure period, classic equations (1) and (2) would produce parameters of $\mu_X = 200$ units/cycle and $\sigma_X = 147.82$ units/cycle for the base case. Decreasing lead-time variability (σ_L) from 7 to 4 periods would decrease σ_X from 147.88 to 93.01 units/cycle. Figure 1 overlays the effect of this decrease on the base-case normal (symmetrical) distribution of X. The vertical lines delineate the reorder points (ROP) that would satisfy a 0.60 CSL before and after the decrease.

Figure 2 shows the directly comparable results for the gamma (asymmetrical) distribution. Observe that the normal approximation predicts that the decrease in σ_X would decrease the reorder point and thus safety stock (ROP-μ_X). By contrast, the gamma distribution predicts that the ROP and safety stock would increase.

$$\mu_X = \mu_D * \mu_L \tag{1}$$

$$\sigma_X = \sqrt{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2} \tag{2}$$

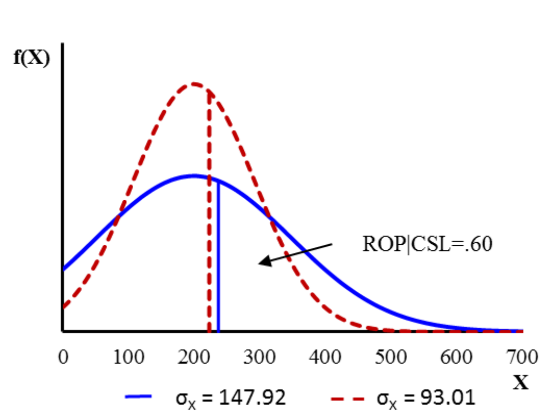

Figure 1. Reorder points for decrease in σ_L from 7 to 4 for normal X

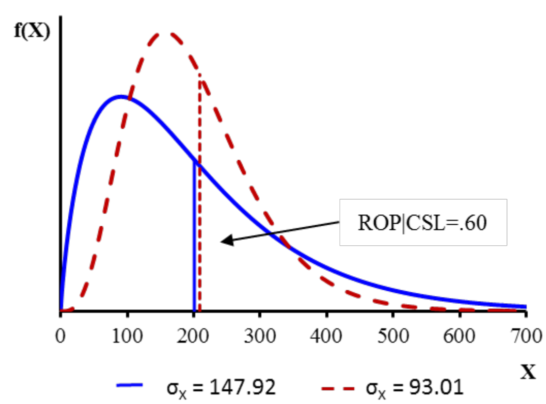

Figure 2. Reorder points for decrease in σ_L from 7 to 4 for gamma X

The foregoing illustration relies on three conditions: (1) a positive-skewed distribution of demand over lead-time, (2) independent μ_L and σ_L, and (3) a relatively low CSL. Empirical evidence indicates that lead-time distributions often lean to the left [4], [7], [11], [15]. These findings are important, for positively skewed lead-time distributions imply that similar attributes would characterize the distribution X. Chopra et al. [3] addressed the low CSL condition by first acknowledging that "most firms aim for fill rates between 97 and 99% (and not cycle service levels)" and then arguing that these fill-rate targets imply CSLs between 50 and 70%. They developed the numerical example in Table 1 to support the argument. This claim and other computational evidence led to the conclusion that, for firms operating at CSLs in the 50 to 70% range, "decreasing lead time is the right lever if they want to cut inventories, not reducing lead time variability."

Table 1. Cycle service Levels and fill rates as a function of reorder points[a]

Reorder Point	Safety Stock	Cycle Service Level	Expected Short	Fill Rate
5000	0	0.500	282.09	0.9718
5040	40	0.523	262.55	0.9737
5080	80	0.545	243.90	0.9756
5120	120	0.567	226.15	0.9774
5160	160	0.590	209.29	0.9791
5200	200	0.611	193.30	0.9807
5240	240	0.633	178.19	0.9822
5280	280	0.654	163.93	0.9836
5320	320	0.675	150.50	0.9850
5360	360	0.695	137.88	0.9862
5400	400	0.714	126.06	0.9874

D is normal(μ_D = 2500, σ_D=500); L=2, Q=10000

X is normal (μ_X =5000, σ_X = 707.11)

Safety stock = ROP - μ_X

Fill rate = 1 - ESC/Q

[a] Adapted from [3]

EXPERIMENTS

The experiments in this research note replicate the best-case computational evidence supporting the foregoing claims under the following best-case conditions: CSL = .60 and L is gamma distributed with μ_L = 10, σ_L = 5. Since the critical condition underlying the significance of the paradox is that CSLs directly correspond to fill rates in the .97 to .99 range, the experiments verified the order quantity (Q) that would achieve at least a .97 fill-rate target. The results (see Table 2) show that decreasing σ_L from 5 to 3 units would increase the ROP and thus safety stock by approximately 3 units. Meanwhile, the Q needed to satisfy a 98% fill rate would decrease substantially from about 1960 to 776 units. The reason is that less uncertainty decreases the expected units short per cycle (ESC) and thus Q in the fill-rate calculation, 1 – ESC/Q.

Consequently, as shown in the last column in Table 2, the sum of safety and cycle stock (Q/2), or the average stock, decreases dramatically from approximately 1,002 units to 388 units.

The key insight is that high fill rates imply low CSLs only when Q is large relative to μ_X as shown in Table 2, where $Q/\mu_X = 2$. The original evidence presented to support the claim that high fill rates imply low CSLs also corroborates this insight. This evidence (see Table 1) is based on Q = 10,000 and μ_X = 5,000 units, which produces $Q/\mu_X = 2$.

Table 2. Replicated and expanded results for lead-time variability experiments

Experiments			Replicated Results		Expanded Results					
Lead Time Process[a]	Mean (μ_L)	Std. Dev. (σ_L)	Safety Stock	Reorder Point (ROP)	Order Qty (Q)	Mean (μ_X)	Ratio (Q/μ_X)	Short / Cycle (ESC)	Short / Yr[b]	Average Stock[c]
Gamma[a]	10	5	20.3	220.3	1,960.4	200	9.8	39.2	146.0	1,002
	10	4	22.0	222.0	1,590.7	200	8.0	31.8	146.0	818
	10	3	22.7	222.7	1,257.9	200	6.3	25.2	146.0	652
	10	2	22.7	222.7	977.1	200	4.9	19.5	146.0	511
	10	1	22.5	222.5	775.9	200	3.9	15.5	146.0	388

Cycle service level (CSL) = .60; fill rate (fr) = .98
[a] discrete approximation
[b] S/Q*ESC, where S = 365 periods/year x μ_D
[c] cycle + safety inventory

CONCLUSIONS

The paradox of lead-time reliability refers to an incongruous increase in safety stock that could result from a decrease in lead-time variability. The enabling conditions include (1) positively skewed lead-time distributions, (2) independent lead-time parameters, and (3) cycle service levels in the 50 to 70% range. Empirical evidence supports the first condition, whereas the last two conditions undermine the practical significance of the paradox. The reason is that they produce an order quantity that is larger than mean demand during lead time. As a result, less uncertainty would decrease cycle stock far more than it would increase safety stock.

Thus, the findings support the following conclusions. First, although the normal approximation may not provide the best characterization of the distribution of demand over the OOS exposure period, it does correctly predict that less lead-time uncertainty will reduce inventories. Second, the central argument supporting the significance of the paradox of lead-time reliability actually undermines its practical significance. Finally, managers should be wary of prescriptions for managing lead-time reliability based solely on changes in safety stock.

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